

# I PUC MID-TERM EXAMINATION, OCT./NOV.-2024

SUBJECT : MATHEMATICS (35)

Time : 3 Hours

Max Marks : 80

## Instructions :

- 1) The question paper has Five parts namely A, B, C, D and E. Answer all the parts.
- 2) PART-A has 15 MCQ's. 5 Fill in the blanks of one mark each.

## PART-A

### I Answer ALL the multiple choice questions : 15x1=15

- 1) If  $X = \{a, b, c, d\}$  and  $Y = \{f, b, d, g\}$  then  $X - Y =$   
A)  $\{a, b\}$       B)  $\{a, c\}$       C)  $\{f, g\}$       D)  $\{b, d\}$
- 2) The relation between the sets  $A = \{2, 3\}$  and  $B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$  is  
A)  $A = B$       B)  $A \neq B$       C)  $A' = B'$       D)  $A' \neq B'$
- 3) Let  $A = \{2, 3\}$ ,  $B = \{1, 5\}$  then the number of subsets of  $(A \times B)$  is  
A) 4      B) 8      C) 16      D) 32
- 4) Statement 1 : A relation  $f : A \rightarrow B$  is said to be a function if every element of set A need not have one and only one image in set B.  
Statement 2 : A function  $f$  is a relation from a non-empty set A to B such that the domain of  $f$  is A and no two distinct ordered pairs in  $f$  have the same first element.  
A) Statement 1 and 2 are true  
B) Statement 1 is true and 2 is false  
C) Statement 1 and 2 are false  
D) Statement 1 is false and 2 is true
- 5) The radian measure of  $240^\circ$  is  
A)  $\frac{4\pi}{3}$       B)  $\frac{3\pi}{4}$       C)  $\frac{2\pi}{3}$       D)  $\frac{7\pi}{3}$
- 6) If  $\tan x = -\frac{5}{12}$ ,  $x$  lies in second quadrant then  $\operatorname{cosec} x$  is  
A)  $\frac{5}{13}$       B)  $\frac{13}{5}$       C)  $\frac{5}{12}$       D)  $\frac{12}{5}$
- 7) The real part of  $(5 - 3i)^3$  is  
A) 15      B) -10      C) 10      D) 27
- 8) The simplified form of  $(-5i)\left(\frac{i}{8}\right)$  is  
A)  $\frac{5}{8}$       B) 0      C)  $\frac{8}{5}$       D) 1
- 9) Match the following  

| A                             | B                                  |
|-------------------------------|------------------------------------|
| i) Slope-point form           | a) $\frac{x}{a} + \frac{y}{b} = 1$ |
| ii) Slope intercept form      | b) $y = mx + c$                    |
| iii) Intercept form           | c) $y - y_1 = m(x - x_1)$          |
| A) (i)-(a) (ii)-(c) (iii)-(b) | B) (i)-(b) (ii)-(a) (iii)-(c)      |
| C) (i)-(b) (ii)-(c) (iii)-(a) | D) (i)-(c) (ii)-(b) (iii)-(a)      |

(P.T.O.)

- 10) If  $7x + 3 < 5x + 9$  then  
 A)  $x > 3$  B)  $x < 6$  C)  $x < 3$  D)  $x > 6$
- 11) The slope of the line passing through the points  $(-2, 6)$  and  $(4, 8)$  is  
 A) 3 B)  $-\frac{1}{3}$  C)  $\frac{1}{3}$  D)  $\frac{3}{4}$
- 12) The equation of the line through  $(-2, 3)$  with slope  $-4$  is  
 A)  $4x + y + 5 = 0$  B)  $4x - y = 5$  C)  $x - 4y - 5 = 0$  D)  $x + 4y = 5$
- 13) The centre of the circle  $x^2 + y^2 - 4x - 8y = 45$  is  
 A)  $(2, 4)$  B)  $(-2, 4)$  C)  $(2, -4)$  D)  $(-2, -4)$
- 14) The length of the latus rectum of the parabola  $y^2 = -8x$  is  
 A) 8 B)  $-8$  C) 4 D)  $-4$
- 15) The eccentricity of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$   
 A)  $\frac{4}{\sqrt{7}}$  B)  $\frac{\sqrt{7}}{4}$  C)  $-\frac{\sqrt{7}}{4}$  D)  $-\frac{4}{\sqrt{7}}$

**II Fill in the blanks by choosing the appropriate answer from those given in the bracket:**

**5x1=5**

$\left[ 6, \frac{1}{4}, 0, \frac{3}{5}, \sqrt{3}, \phi \right]$

- 16)  $A \cap A' =$  \_\_\_\_\_.
- 17) The value of  $\tan\left(\frac{19\pi}{3}\right)$  is \_\_\_\_\_.
- 18) The radius of the circle  $2x^2 + 2y^2 - x = 0$  is \_\_\_\_\_.
- 19) The length of major axis of the ellipse  $4x^2 + 9y^2 = 36$  is \_\_\_\_\_.
- 20) The distance of the point  $(3, -5)$  from the line  $3x - 4y - 26 = 0$  is \_\_\_\_\_.

**PART-B**

**III Answer any SIX questions :**

**6x2=12**

- 21) If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{2, 4, 6, 8\}$ . Find  $(A - B)$  and  $(B - A)$ .
- 22) If  $t(C) = \frac{9C}{5} + 32$  then find  $t(28)$ .
- 23) The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes.  
 (Use  $\pi = 3.14$ )
- 24) Prove that  $3 \sin\left(\frac{\pi}{6}\right) \sec\left(\frac{\pi}{3}\right) - 4 \sin\left(\frac{5\pi}{6}\right) \cot\left(\frac{\pi}{4}\right) = 1$
- 25) Express  $\frac{5+i\sqrt{2}}{1-i\sqrt{2}}$  in the form  $(a + ib)$
- 26) If  $\left(\frac{1+i}{1-i}\right)^m = 1$ , then find the least positive integral value of  $m$ .



- 27) Solve the inequality  $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$  and show the graph of solution on number line.
- 28) Find all the pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.
- 29) Find the equation of the line parallel to the line  $3x - 4y + 2 = 0$  and passing through the point  $(-2, 3)$

**PART-C**

**IV Answer any SIX questions :**

**6x3=18**

- 30) If  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $A = \{2, 4, 6, 8\}$  and  $B = \{2, 3, 5, 7\}$ , then prove that  $(A \cup B)' = A' \cap B'$
- 31) If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4\}$  and  $C = \{4, 5, 6\}$  then find  $(A \times B) \cap (A \times C)$ .
- 32) Write the relation  $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$  in Roaster form write its Domain and Range.
- 33) Let  $f(x) = x + 1$  and  $g(x) = 2x - 3$  be two functions over the set of non-negative real numbers. Find  $(f + g)(x)$ ,  $(f - g)(x)$  and  $\left(\frac{f}{g}\right)(x)$ .
- 34) Prove that  $\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{(1 + \tan x)^2}{(1 - \tan x)^2}$ .
- 35) Prove that  $\cos(3x) = 4\cos^3x - 3\cos x$
- 36) Ravi obtained 70 and 75 marks in first two test. Find the minimum marks he should get in the third test to have an average of atleast 60 marks.
- 37) Derive the formula to find the equation of the line formed by the points  $(x_1, y_1)$  and  $(x_2, y_2)$ .
- 38) Find the equation of the circle which passes through the points  $(2, -2)$  and  $(3, 4)$  and whose centre lies on the line  $x + y = 2$ .

**PART-D**

**V Answer any FOUR questions :**

**4x5=20**

- 39) Define greatest integer function. Draw the graph of the function. Write the domain and range of the function.
- 40) Define modulus function. Draw the graph of the function. Write the domain and the range of the function.
- 41) Prove that  $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\left(\frac{x+y}{2}\right)$ .
- 42) Derive the equation of a line with x-intercept and y-intercept a and b respectively. Also find the equation of the line, which makes intercepts  $-3$ , and  $2$  on the x and y-axes respectively.
- 43) Define parabola and find the equation of parabola in its standard form  $y^2 = 4ax$ . Also find the equation of the parabola with directrix  $x = -2$  and focus  $(2, 0)$ .
- 44) Find the coordinates of foci, the length of major and minor axes and the eccentricity of the ellipse  $9x^2 + 4y^2 = 36$ .

- 45) Derive the formula to find the length of the perpendicular from the point  $(x_1, y_1)$  to a line

$$Ax + By + C = 0 \text{ in the form } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

**PART-E**

**VI Answer the following questions :**

- 46) Prove geometrically that  $\cos(x + y) = \cos x \cos y - \sin x \sin y$  and hence show that  $\cos 2x = \cos^2 x - \sin^2 x$ .

**OR**

Define Hyperbola and derive the equation of the hyperbola in the form  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . (6)

- 47) Prove that  $\tan 3x \cdot \tan 2x \cdot \tan x = \tan 3x - \tan 2x - \tan x$ .

**OR**

The line through the points  $(h, 3)$  and  $(4, 1)$  intersects the line  $7x - 9y - 19 = 0$  at right angle. Find the value of  $h$ . (4)

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