II PUC MID-TERM EXAMINATION, OCT./NOV.-2024

SUBJECT : MATHEMATICS (35)

Time : 3 Hours **Max Marks : 80**

Instructions :

1) **The question paper has five parts namely A, B, C, D and E. Answer all the parts.**
2) **Part A has 15 Multiple choice questions, 5 Fill in the blanks of 1 mark each.**
3) **Use the graph sheet for question on linear programming problem in Part-E.**

PART - A

I Answer ALL the multiple choice questions : **15x1=15**

1) The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is
A) Reflexive B) Symmetric C) Transitive D) Equivalence relation

2) The modulus function $f : R \to R$ given by $f(x) = |x|$ is
A) one-one and onto B) one-one but not onto
C) onto but not one-one D) neither one-one nor onto

3) The principal value of $\cos^{-1}\left(-\sqrt{3}/2\right)$ is
A) $\pi/6$ B) $\pi/3$ C) $5\pi/6$ D) $2\pi/3$

4) Match the following :

A	B
a) Domain of $\sec^{-1}x$	i) $\left(-\pi/2, \pi/2\right)$
b) Domain of $\sin^{-1}x$	ii) $R-(-1,1)$
c) Range of $\tan^{-1}x$	iii) $[-1, 1]$

A) a-(ii) b-(iii) c-(i) B) a-(iii) b-(i) c-(ii) C) a-(iii) b-(ii) c-(i) D) a-(ii) b-(i) c-(iii)

5) If a matrix has 24 elements, then the total number of possible matrices of different order are
A) 8 B) 6 C) 4 D) 2

6) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to
A) 6 B) ± 6 C) -6 D) 0

7) The point of discontinuity of the function $f(x) = 1/x,\ \forall x \in R$, is
A) $x = 1$ B) $x = 0$ C) $x = 2$ D) $x = -1$

8) If $y = \log(\log x)$, $x > 1$, then dy/dx is
A) $\dfrac{1}{x\log x}$ B) $\dfrac{1}{\log x}$ C) $\dfrac{1}{\log(\log x)}$ D) $\dfrac{x}{\log x}$

9) Let I be an interval contained in the domain of a real valued function and then
Statement 1 : f is said to be increasing on I, if $x_1 < x_2$ is $I \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$
Statement 2 : f is said to be decreasing on I, if $x_1 > x_2$ in $I \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$
A) Statement 1 is false statement 2 is true B) Statement 1 is true but statement 2 is false
C) Both statement 1 and 2 are false D) Both statement 1 and 2 are true

10) The total revenue in Rs. received from the sale of x units of product is given
$R(x) = 13x^2 + 26x + 15$; then the marginal revenue when $x = 7$ is Rs.
A) 208 B) 108 C) 308 D) 280

(P.T.O.)

11) The direction cosines of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ are

A) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$ B) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ C) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ D) $\frac{1}{\sqrt{6}}, 1/\sqrt{6}, 2/\sqrt{6}$

12) The unit vector in the direction of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ is

A) $\frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}}$ B) $\frac{2\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{14}}$ C) $\frac{-2\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{14}}$ D) $\frac{2\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{14}}$

13) The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is

A) $10/\sqrt{6}$ B) $7/\sqrt{6}$ C) $\sqrt{6}/10$ D) $5/\sqrt{6}$

14) If a line makes angles 90°, 135°, 45° with X, Y and Z axes respectively, then direction cosines are

A) $1, -1/\sqrt{2}, 1/\sqrt{2}$ B) $0, -1/\sqrt{2}, 1/\sqrt{2}$ C) $0, -1/\sqrt{2}, -1/\sqrt{2}$ D) $0, -1/2, 1/2$

15) The angle between pair of lines given by $\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = (5\hat{i} - 2\hat{j}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ then θ is

A) $\cos^{-1}(1/2)$ B) $\cos^{-1}\left(\frac{21}{19}\right)$ C) $\cos^{-1}\left(\frac{19}{21}\right)$ D) $\cos^{-1}(9/21)$

II Fill in the blanks by choosing the appropriate answer from those given in the bracket:
$(\pi/3,\ 0,\ 3/4,\ \pi/4,\ 3,\ 12\pi)$ **5x1=5**

16) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$ then the value of α is ________.

17) If value of $\tan^{-1}\left(2\cos\left(2\sin^{-1}(1/2)\right)\right)$ is = ________.

18) The function $f(x) = kx^2$ if $x \leq 2$ and $f(x) = 3$ if $x > 2$ is continuous at $x = 2$, then the value of k is ________.

19) The rate of change of area of a circle with respect to its radius at r = 6 cms is ________ .

20) The value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{k}\times\hat{i}) + \hat{k}\cdot(\hat{i}\times\hat{j})$ ________ is.

PART -B

III Answer any SIX questions : **6x2=12**

21) Show that the relation in the set of real numbers R defined as R = {(a, b) : a ≤ b} is reflexive and transitive.

22) Prove that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)\ x \in \left[-1/2, 1/2\right]$

23) If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ then find the values of x and y.

24) Find the area of the triangle with vertices (2, 7), (1, 1) and (10, 8) using determinant method.

25) If $2x + 3y = \sin y$ then find $\frac{dy}{dx}$

26) Find the point of minima for the function $f(x) = 9x^2 + 12x + 2$.

27) The radius of a circle is increasing at the rate of 0.7 cm /sec. What is the rate of increase of its circumference.

28) If for a unit vector $\vec{a}$ and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 12$ find $|\vec{x}|$.

29) Find the distance between the lines l_1 and l_2 given by $\vec{r} = \hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r} = 3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

PART-C

V Answer any SIX questions : **6x3=18**

30) Let L be the set of all lines in plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation.

31) Express the following function $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x| < a$ in the simplest form.

32) If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$ find fog and gof, also show that fog $\neq$ gof.

33) If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Show that $F(x)\cdot F(y) = F(x + y)$

34) Differentiate $x^{\sin x}$, $x > 0$ with respect to x.

35) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ then find $\frac{dy}{dx}$.

36) Find the intervals in which the function f, given by $f(x) = x^2 - 4x + 6$ is
a) increasing
b) decreasing

37) If $\vec{a} = 5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b} = \hat{i}+3\hat{j}-5\hat{k}$ then show that the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular.

38) Find the cosine of the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$

PART - D

VI Answer any FOUR questions : **5x4=20**

39) State whether the function $f : R \to R$ defined by $f(x) = 1 + x^2$ is one-one, onto or bijective. Justify your answer.

40) Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $y = \{y \in N : y = 4x + 3, \text{ for some } x \in N\}$. Show that f is invertible and hence find the inverse of f.

41) If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. Calculate AC, BC and (A + B) C, verify that (A + B)C = AC + BC.

(P.T.O.)

42) If $A=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$ then verify that $(A-B)'=A'-B'$.

43) Solve the following system of equations by matrix method :
$2x+3y+3z=5,\quad x-2y+z=-4,\ 3x-y-2z=3.$

44) If $y=Ae^{mx}+Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+(mn)y=0$

45) If $y=\left(\tan^{-1}x\right)^2$ show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$.

PART-E

VII Answer the following questions : (6 + 4=10)

46) Maximize $Z=3x+2y$ subject to constraints;
$x+2y\le 10,\ 3x+y\le 15,\ x\ge 0,\ \ y\ge 0$ by graphical method.

OR

Solve the following problem graphically Minimize and Maximize $z=5x+10y$ subject to
$x+2y\le 120,\ x+y\ge 60,\ x-2y\ge 0\quad x\ge 0\ \ y\ge 0$ **(6)**

47) If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ show that $A^2-5A+7I=0$ where I is 2×2 identity matrix and O is 2×2 zero matrix. Using this equation, find A^{-1}.

OR

Find the values of k if

$f(x)=\begin{cases}kx+1 & \text{if } x\le\pi\\ \cos x & \text{if } x>\pi\end{cases}$ is continuous at $x=\pi$. **(4)**

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